Discrete time Fourier transform (DTFT)

if $x[n]$ is absolutely summable

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty$$

then the DTFT of $x[n]$ is given by.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad \ldots\ldots (1)$$

The inverse DTFT is given by.

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

DTFT transforms discrete time sequence $x[n]$ into a continuous function $X(e^{j\omega})$ of real variable ω.

Note Ω : analog omega

ω : digital omega

Note $X(e^{j\omega})$ is a periodic function (2π)

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

This is the DTFT of $h[n]$

midterm feb 28.

EX Determine the DTFT of:

$$x[n] = 0.5^n u[n]$$

SOL

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} 0.5^n u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} 0.5^n e^{-j\omega n}$$

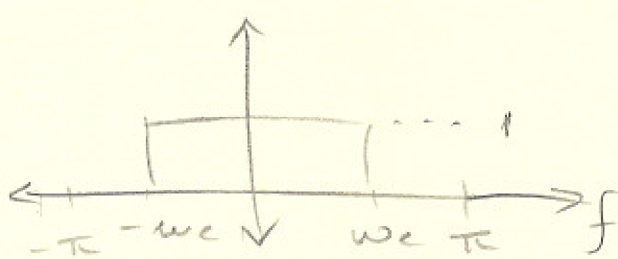
$$= \sum_{n=0}^{\infty} (0.5 e^{-j\omega})^n$$

$$= \frac{1}{1-0.5e^{-j\omega}}$$

$$= \frac{e^{j\omega}}{e^{j\omega}-0.5}$$

EX Determine the impulse response of an ideal low pass filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < |\omega| < \pi. \end{cases}$$



SOL
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\, d\omega$$

$$= \frac{\left[e^{j\omega n}\right]_{-\omega_c}^{\omega_c}}{2\pi j n}$$

$$= \frac{e^{j\omega_c n} - e^{j\omega_c n}}{2\pi j n}$$

$$= \frac{\sin \omega_c n}{\pi n}$$

$$= \frac{\omega_c}{\pi} \operatorname{sinc}(\omega_c n)$$

It should be noted that this is a non-causal and hence not implemented in real time

1.8.2 Properties of FT

A) Linearity

$$\mathcal{F}\{a x_1[n] + b x_2[n]\}$$

$$= a X_1(e^{j\omega}) + b X_2(e^{j\omega})$$

B) Time Shifting

$$\mathcal{F}\{x[n-k]\} = e^{-j\omega k} X(e^{j\omega})$$

c) Freq Shifting.

$$\mathcal{F}\{e^{j\omega_0 n} x[n]\} = X(e^{j(\omega-\omega_0)})$$

D) Folded Sequence

$$\mathcal{F}\{x[-n]\} = X(e^{-j\omega})$$

$$= X^*(e^{j\omega})$$

E) Differentiation of freq.

$$\mathcal{F}\{n x[n]\} = j\frac{dX}{d\omega}(e^{j\omega})$$

F) Convolution

$$\mathcal{F}\{x[n] * y[n]\}$$

$$= X(e^{j\omega}) Y(e^{j\omega})$$

G) Multiplycation

$$\mathcal{F}\{x[n]\,y[n]\} = X(e^{j\omega}) * Y(e^{j\omega})$$

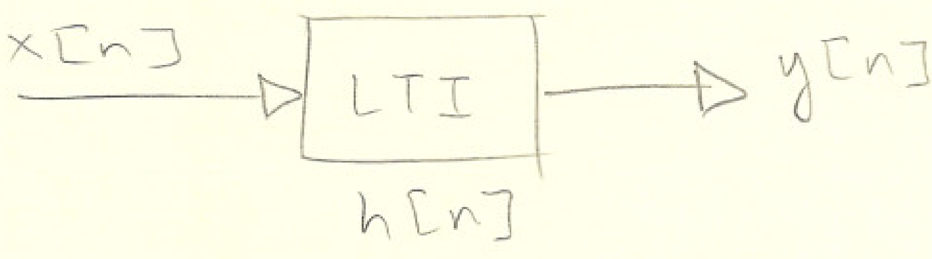
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, Y(e^{j(\omega-\theta)})\, d\theta$$

h) The energy of a sequence $x[n]$

$$\left.\begin{aligned} E_x &= \sum_{n=-\infty}^{\infty} |x[n]|^2 \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2\, d\omega \end{aligned}\right\} \text{Parseval's relation}$$

Note: Table 2.3 will be provided for exam

1.8.2 Responce of an arbutrary Seq in the freq domain.



$$y[n] = x[n] * h[n]$$

$$Y(e^{j\omega}) = X(e^{j\omega})\, H(e^{j\omega})$$

Proof

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k] h[n-k] \right) e^{-j\omega n}$$

let $n-k = m \quad \therefore \quad n = m+k$

$$= \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k] h[m] e^{-j\omega(m+k)}$$

$$= \left(\sum_{m=-\infty}^{\infty} h[m] e^{-j\omega m} \right) \left(\sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k} \right)$$

$$= X(e^{j\omega}) H(e^{j\omega})$$

QED